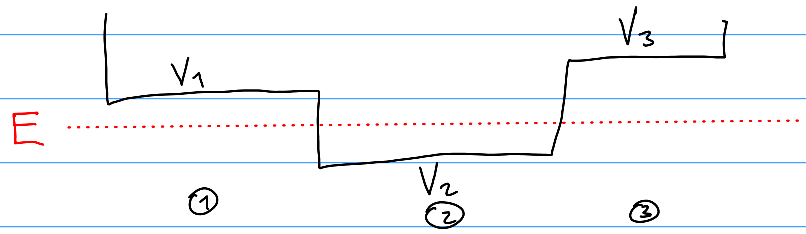
Recapitulación

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(\vec{r})\right]\varphi(x) = E\varphi(x)$$



Para $E > V_j$: $E - V_k = \frac{\hbar^2 k_j^2}{2m}$ $\quad \varphi_j(x) = A_j e^{ik_jx} + A_j' e^{-ik_jx}$

Para $E < V_j$: $V_k - E = \frac{\hbar^2 \rho_j^2}{2m}$ $\quad \varphi_j(x) = B_j e^{\rho x} + B_j' e^{-\rho x}$

Para $E = V_j$: $\quad \varphi_j(x) = Cx + D$

Un resultado útil.

E debe ser > min(V(x)) $\qquad$ complement CT III M_{III}

$\hat{H} = \hat{T} + \hat{V}$ $\qquad$ si $\hat{H}|\varphi\rangle = E|\varphi\rangle$

mult por $\langle\varphi|$

$\langle\varphi|\hat{H}|\varphi\rangle = E$ ~~$\langle\varphi|\varphi\rangle$~~ 1

$\langle\hat{T}\rangle + \langle\hat{V}\rangle = E$

$\langle T\rangle = \frac{1}{2m}\langle\hat{P}^2\rangle = \int p^2 |\varphi(p)|^2 dp \geq 0$

$\langle V\rangle = \int V(x)|\varphi(x)|^2 dx \geq \int(-V_0)|\varphi(x)|^2 dx \geq -V_0 = (\min V(x))$

$E = \langle\hat{T}\rangle + \langle\hat{V}\rangle \geq \langle\hat{V}\rangle \geq -V_0$

Comentar que $E_{min} > \min(V(x))$

Pozo cuadrado

CT I H_I

2-c. Bound states: square well potential

α. *Well of finite depth*

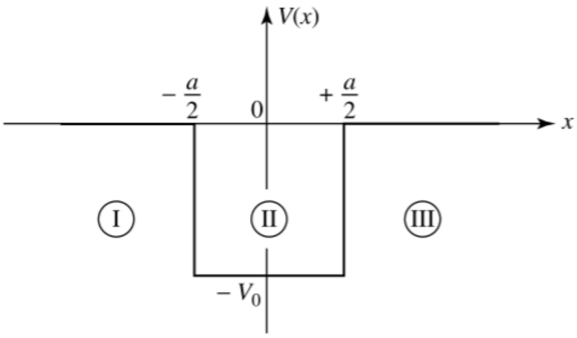


Figure 4: Square well potential.

We shall limit ourselves to studying the case $-V_0 < E < 0$ (the case $E > 0$ was included in the calculations of the preceding section 2-b-α).

In regions I $\left(x < -\frac{a}{2}\right)$, II $\left(-\frac{a}{2} \leqslant x \leqslant \frac{a}{2}\right)$, and III $\left(x > \frac{a}{2}\right)$ shown in Fig. 4, we have respectively:

$$\varphi_{\mathrm{I}}(x) = B_1\, \mathrm{e}^{\rho x} + B_1'\, \mathrm{e}^{-\rho x} \tag{36-a}$$
$$\varphi_{\mathrm{II}}(x) = A_2\, \mathrm{e}^{ikx} + A_2'\, \mathrm{e}^{-ikx} \tag{36-b}$$
$$\varphi_{\mathrm{III}}(x) = B_3\, \mathrm{e}^{\rho x} + B_3'\, \mathrm{e}^{-\rho x} \tag{36-c}$$

with

$$\rho = \sqrt{-\frac{2mE}{\hbar^2}} \tag{37}$$

Queda encontrar la relación entre ρ, k y las otras constantes para satisfacer las condiciones de frontera.

$$k = \sqrt{\frac{2m(E+V_0)}{\hbar^2}} \tag{38}$$

Since $\varphi(x)$ must be bounded in region I, we must have:

$$B_1' = 0 \tag{39}$$

The matching conditions at $x = -\frac{a}{2}$ then give:

$$A_2 = \mathrm{e}^{(-\rho+ik)a/2}\,\frac{\rho+ik}{2ik}\,B_1$$
$$A_2' = -\,\mathrm{e}^{-(\rho+ik)a/2}\,\frac{\rho-ik}{2ik}\,B_1 \tag{40}$$

and those at $x = a/2$:

$$\frac{B_3}{B_1} = \frac{e^{-\rho a}}{4ik\rho}\left[(\rho + ik)^2\, e^{ika} - (\rho - ik)^2\, e^{-ika}\right]$$

$$\frac{B_3'}{B_1} = \frac{\rho^2 + k^2}{2k\rho}\sin ka \tag{41}$$

But $\varphi(x)$ must also be bounded in region III. Therefore, it is necessary that $B_3 = 0$, that is:

$$\left(\frac{\rho - ik}{\rho + ik}\right)^2 = e^{2ika} \tag{42}$$

Since ρ and k depend on E, equation (42) can only be satisfied for certain values of E. Imposing a bound on $\varphi(x)$ in all regions of space thus entails the quantization of energy. More precisely, two cases are possible:

(i) if:

$$\frac{\rho - ik}{\rho + ik} = -\, e^{ika} \tag{43}$$

we have:

$$\frac{\rho}{k} = \tan\left(\frac{ka}{2}\right) \tag{44}$$

$$\frac{\rho - ik}{\rho + ik} = -e^{ika} = -\cos ka - i \operatorname{sen} ka$$

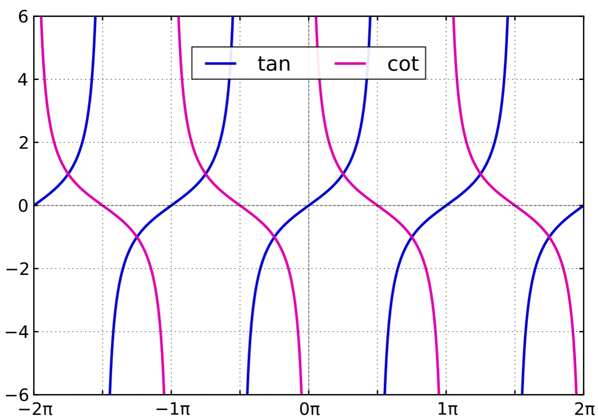
$$\frac{\rho - ik}{\rho + ik}\,\frac{\rho - ik}{\rho - ik} = \frac{\rho^2 - 2ik\rho - k^2}{\rho^2 + k^2} \Rightarrow +\cos ka = \frac{-\rho^2 + k^2}{\rho^2 + k^2}$$

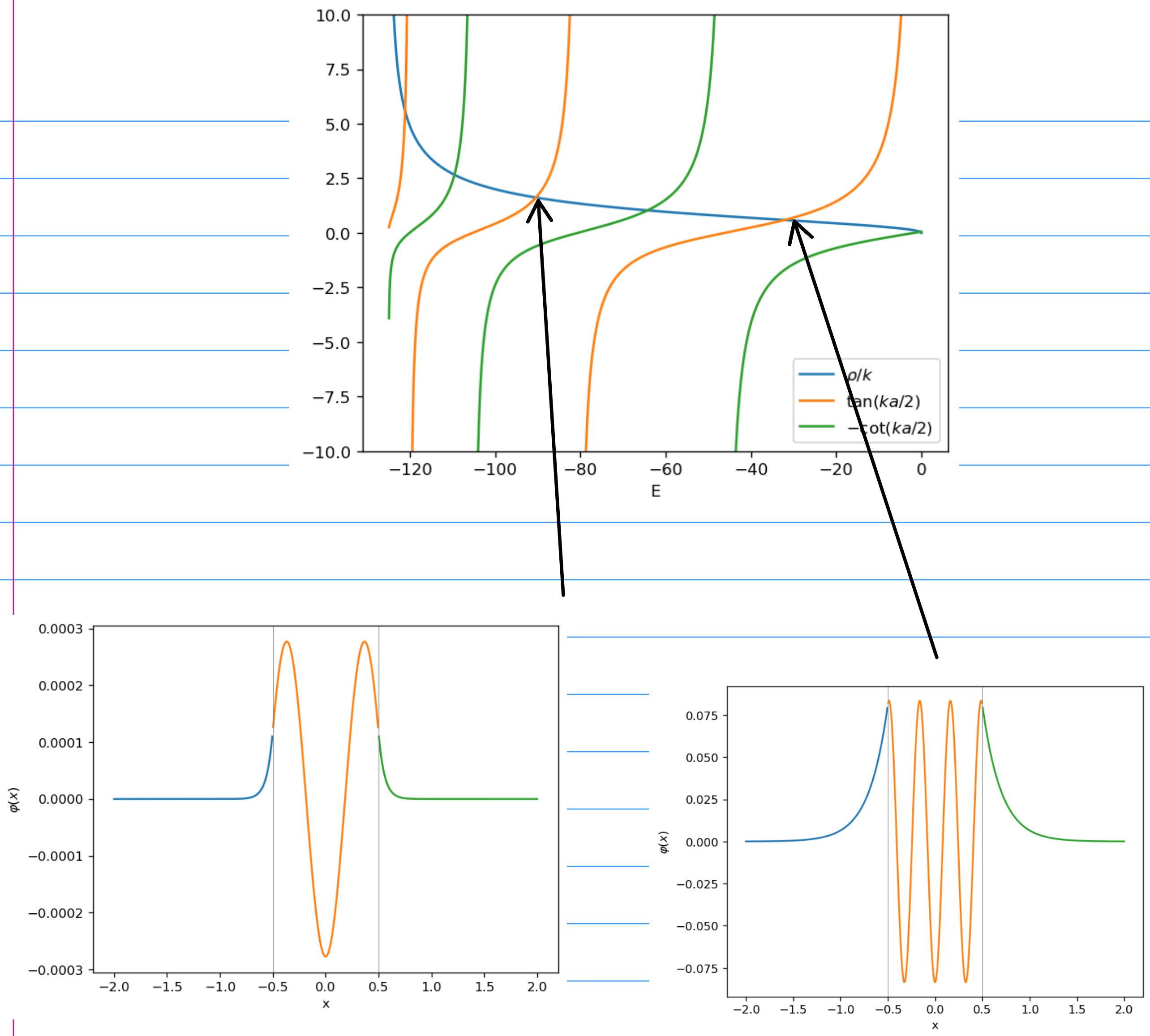
$$+\operatorname{sen} ka = +\frac{2k\rho}{\rho^2 + k^2}$$

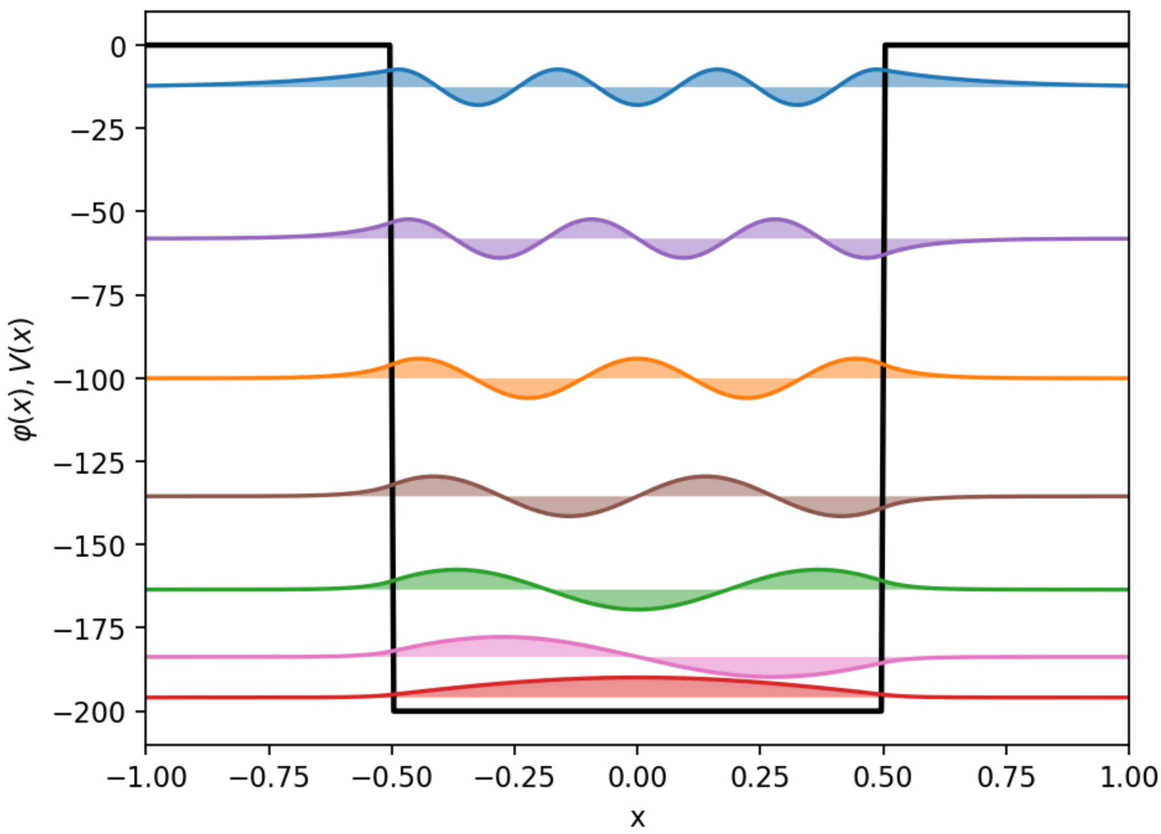
$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\tan ka = \frac{2k\rho}{k^2 - \rho^2} = \frac{2\,\rho/k}{1 - \rho^2/k^2} \Rightarrow \tan\left(\frac{ka}{2}\right) = \rho/k$$

Análogo $\quad \frac{\rho - ik}{\rho + ik} = e^{ika} \Rightarrow -\cot\left(\frac{ka}{2}\right) = \rho/k$




10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−120
−100
−80
−60
−40
−20
0
E
ρ/k
tan(ka/2)
−cot(ka/2)
0.0003
0.0002
0.0001
0.0000
−0.0001
−0.0002
−0.0003
φ(x)
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
x
0.075
0.050
0.025
0.000
−0.025
−0.050
−0.075
φ(x)
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
x


0
−25
−50
−75
−100
−125
−150
−175
−200
φ(x), V(x)
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x